KAKATIYA GOVERNMENT COLLEGE

HANUMAKONDA

Name : Dr. M. Venugopal

Designation : Assistant Professor of Mathematics

Year of Award of PhD. : 2022

Name of the University : Kakatiya University

Year of entering into Govt. Service : 2013.

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Hanamkonda.

Signature

Name & Designation M. Venugopal
Asst Prof of Maths

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Name & Designation M. Venugopal
Asst Prof of Maths

Kakatiya University

4210

No. 0926

PROVISIONAL CERTIFICATE

Ph.D.

This is to Certify that M. Venu Gopal

Son/Daughter of M. Muthanna has been declared qualified

for the award of the Ph.D. Degree in Mathematics of this

University in May, 2022.

Topic of Thesis:

"STUDY OF DISPERSIVE BEHAVIOR OF WAVES IN POROELASTIC LAYERS AND HALF SPACES"

Warangal T.S. - 506 009

Date: 27-05-2022

for Registrar

EXAMINATION BRANCH
KAKATIYA UNIVERSITY
WARANGAL – 506 009 (TS) INDIA

No. 436 /Ph.D./E1/KU/2022 Date: 16-05-2022

PRESS NOTE

Mr/Ms. **M. Venu Gopal**, Research Scholar in **Mathematics,** Kakatiya University, Warangal, who has presented the thesis entitled **"STUDY OF DISPERSIVE BEHAVIOR OF WAVES IN POROELASTIC LAYERS AND HALF SPACES"** has been declared qualified for the Degree of **Doctor of Philosophy** (Ph.D.) in **Mathematics** of Kakatiya University.

"By Order"

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2. The Secretary, University Grants Commission, New Delhi-110 002.
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5. The Coordinating Officer, U.G.C. Unit, Kakatiya University, Warangal.
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17. The Person concerned **(M. Venu Gopal S/D/o. M. Muthanna)**

(4210)

OFFICE OF THE DEAN
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Kakatiya University, Warangal – 506 009 (T.S.), India
(Accredited with "A" Grade by the NAAC)

Prof. K.David
Dean
Professor of Geology

Phone: (O) 0870 – 2461434

No.20 /DFS/KUW/2021

Date: 16-03-2021

ORDER

Sub: Faculty of Sciences - Ph. D. Programme in Department of Mathematics – for the Academic year 2015-2016 - Regarding

Ref: 15/DFS/KUW/2018 dated 09/02/2018

***

On the recommendation of the Admission Committee and with the approval of the Vice-Chancellor, Kakatiya University, Warangal, the following candidates has been provisionally selected for admission to the Ph. D. Programme for the year 2015-2016 in the Department of Mathematics.

SL NO	Name of Cadidates	Social Statues	Title of the Ph.D Research topic	Name of the Research Supervisor	Research
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3	Aalla Ravi Kumar	BC-D	Some problems of wave propagation in micro-elastic solids	Dr. K.Somaiah	Part-Time
4	Kumar Ragula	BC-B	Numerical methods for a class of singularly perturbed differential – difference equations	Dr.B.S.L.Soujanya G.	Part-Time
5	Dolledla Harika	OC	Investigation of steady state probability vector of positive definite regularized linear systems of stochastic matrices	Dr.L.P.Raj Kumar	Part-Time
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9	T.Thirupathi	BC-D	Performance analysis of multi server queuing systems with disaster	Dr.L.P.Raj Kumar	Part-Time
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11	A.Satyanarayana	BC-B	Study on estimating the reliability for stress-strength models	Dr.M.Tirumala Devi	Part-Time

12	Yakoobpasha.MD	BC-B	Reliability to develop the stress dependent reliability models	Dr.T.Sumathi Uma Maheswari	Part-Time
13	Sandhya Rani Kanukuntla	BC-B	Estimation of reliability on stochastic stress-strength models	Dr.T.Sumathi Uma Maheswari	Part-Time
14	Gonela Anitha	BC-B	Propagation of waves in rotating micro elastic solids	Dr.K.Somaiah	Part-Time
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16	Kondam Ravinder Reddy	OC	Mathematical modeling of retrieval queueing system under fuzzy environment	Dr.L.P.Raj Kumar	Part-Time
17	Yakaiah.K	BC-A	A Study on the coefficient inequalities for some subclasses of analytic functions	Dr.R.Bharavi sharma	Part-Time
18	Chepuri Rajitha	SC	Performance study on the dependent queueing system with server on vacation	Dr.L.P. Raj Kumar	Part-Time
19	Venkateswara Rao Burugu	SC	Studies of wave propagation problems in elastic solids	Dr. K. Somaiah	Part-Time
20	Narsimhara Rao Kandula	SC	Wave propagation in different elastic solids	Dr. K. Somaiah	Part-Time
21	Gouraveni Saritha	ST	Reliability estimation for stress-strength models	Dr.M.Tirumala Devi	Full-Time
22	Sameena Afreen	BC-E	Study on survival Analysis	Dr.M.Tirumala Devi	Part-Time
23	Poonem Latha madhuri	ST	Study of wave propagation on anisotropic poro elastic solids using biot extension theory	Prof.P.Malla Reddy	Part-Time
24	M.Jyothirmai	BC-C	Performance study of priority based internet router with self similar input traffic transient queueing system markovian modelled input process	Prof.P.Malla Reddy	Part-Time

24/03/23.
DEAN

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4. The Supervisors concerned
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KAKATIYA UNIVERSITY
WARANGAL - 506 009 (T.S.)
MEMORANDUM OF MARKS

B 5095716

Examination PRE-Ph. D. (MATHEMATICS) AUG., 2019   Date : 04-11-2019

Candidate's Name M VENU GOPAL

Father's Name M MUTHANNA   Roll No. 170002606

SUBJECT	MAXIMUM MARKS	PASS MARKS	MARKS SECURED	SUBJECT RESULT
RESEARCH METHODOLOGY	100	050	073	PASS
CONTINUUM MECHANICS & PARTIAL DIFFR. EQU.	100	050	083	PASS

TOTAL MARKS : ONE HUNDRED AND FIFTY SIX   156

RESULT : PASSED - - - - - - - - - - - - - -

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DEPARTMENT OF MATHEMATICS

University College, Kakatiya University, Warangal -506 009.Telangana
9849133398, 0870-2461425 |Office| Email: lp-raj8@yahoo.com.

CERTIFICATE

This is to certify that Sri/~~Smt./Kumari~~. **M. Venu Gopal** working for his/her Ph.D. (~~Full-Time~~/part-Time) on the topic *"Wave propagation in poroelastic solids"* under the supervision of Dr./Prof. **P. Malla Reddy** has presented his/her **Seminar I/~~II~~**, on the topic with title ***"Wave propagation in poroelastic solids"*** as per UGC regulation-2009 of Doctor of Philosophy conducted by the Department of Mathematics, Kakatiya University, Warangal on **31-10-2020 at 10.00 a.m**.

P.M. Reddy

Prof. P. Malla Reddy
Professor
Department of Mathematics
Kakatiya University
Warangal.

HEAD
Department of Mathematics,
Kakatiya University, Warangal

HEAD
Department of Mathematics
KAKATIYA UNIVERSITY
WARANGAL - 506 009 T.S

Chairperson, BoS
Department of Mathematics,
Kakatiya University, Warangal

Chairperson, BOS
Department of Mathematics
University College
Kakatiya University, Warangal.

31.10.2020
DEAN
Faculty of Science
Kakatiya University, Warangal

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Kakatiya University, Warangal – 506 009 (T.S.), India
(Accredited with "A" Grade by the NAAC)

Prof. K.David
Dean
Professor of Geology

Phone: (O) 0870 – 2461434

No.65/DFS/KUW/2021

Date: 12/11/2021

ORDERS

Sub: Faculty of Science – Department of Mathematics Mr.M.Venu Gopal Scholar – Change of title of thesis approved – Orders – issued.

Ref: 1. Application of the Research Scholar dated 21-09-2021 duly forwarded by the supervisor, Head of the Department and the Chairperson, BOS concerned.
2. KU Orders No.15/DFS/KUW/2018 dated 09-02-2018
3. KU Orders No.20/DFS/KUW/2021 dated 16-03-2021

* * *

The Vice-Chancellor, Kakatiya University, Warangal is pleased to accord permission to **Mr.M.Venu Gopal**, Research Scholar, Department of Mathematics, Kakatiya University for the change of title of his Ph D. thesis from

"WAVE PROPAGATION IN POROELASTIC SOLIDS."

TO

" STUDY OF DISPERSIVE BEHAVIOR OF WAVES IN POROELASTIC LAYERS AND HALF SPACES."

DEAN

To
Mr. M. Venu Gopal,
Research Scholar,
Department of Mathematics,
Kakatiya University.

Copy to:
1. The Principal, University College, KU
2. The Head, Department of Mathematics, KU
3. The Chairperson, Board of Studies in Mathematics, KU
4. Prof.P.Malla Reddy, (Supervisor), Department of Mathematics, KU
5. The Controller of Examinations, KU

DEPARTMENT OF MATHEMATICS

University College, Kakatiya University, Warangal -506 009.Telangana

✆9849133398, ☎0870-2461425 [Office] *Email: lp-raj8@yahoo.com.*

CERTIFICATE

This is to certify that Sri. **M. Venugopal** working for his Ph.D. (Part-Time) on the topic *"Study of Dispersive Behavior of Waves in Poroelastic Layers and Half Spaces"* under the supervision of **Prof. P. Malla Reddy** has presented his Seminar II, as per UGC regulation-2009 of Doctor of Philosophy conducted by the Department of Mathematics, University College, Kakatiya University, Warangal on **15-12-2021 at 11.00 a.m.**

Supervisor Department of Mathematics, Kakatiya University, Warangal	HEAD Department of Mathematics, Kakatiya University, Warangal	Chairperson, BoS Department of Mathematics, Kakatiya University, Warangal	DEAN Faculty of Science Kakatiya University, Warangal

Supervisor Details

Name: Prof. P. Malla Reddy

Department: Mathematics

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Prof. Y. Prameela Devi
Professor of Zoology & Dean

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Date: 09/02/2018

ORDERS

Sub: Faculty of Science - Ph. D. Admissions for the Year 2015-16 and 2016-17 - Department of Mathematics - Orders – Issued

* * *

On the recommendation of the Admission Committee and with the approval of the Vice-Chancellor, Kakatiya University, Warangal the following candidates have been provisionally selected for admissions into the Ph.D. Programme for the year 2015-16 and 2016-17 in the Department of Mathematics.

Sl No	Name of the Candidate	Social Status	Name of the Research Supervisor
01.	Sambasiva Rao. S.	OC	Dr. R. Bharavi Sharma
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03.	Aalla Ravi Kumar	BC-D	Dr. K. Somaiah
04.	Kumar Ragula	BC-B	Dr. B.S.L. Soujanya. G
05.	Dollebla Harika	OC	Dr. L.P. Raj Kumar
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08.	Laxman Kumar. T	OC	Prof. P. Malla Reddy
09.	T. Thirupathi	BC-D	Dr. L.P. Raj Kumar
10.	Swarnakar Dornala	OC	Dr. B.S.L. Soujanya. G
11.	Satyanarayana. A	BC-B	Dr. M. Thirumala Devi
12.	Balla Prashanthi	BC-B	Dr. M. Thirumala Devi
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14.	Sandhya Rani Kanukuntla	BC-B	Dr. T. Sumathi Uma Maheshwari
15.	Gonela Anitha	BC-D	Dr. K. Somaiah
16.	Ala Sindhuja	BC-D	Prof. P. Malla Reddy
17.	Kondam Ravinder Reddy	OC	Dr. L.P. Raj Kumar
18.	Yakaiah. K.	BC-A	Dr. R. Bharavi Sharma

5. He/She should submit half-yearly progress reports on the work through the Supervisor, Chairperson, Board of Studies and Head of the Department to the undersigned.

6. The candidates should undergo course work and pass Pre-Ph. D. examination within two successive attempts from the date of registration.

7. The in-service candidates working outside Warangal, and registered as Part-time scholars, are required to put in at least six months of attendance in the Department.

8. In all other matters they shall be governed by the existing rules and regulations of the Ph. D. Programme. effective for the Ph. D. admissions in vogue

Any deviation in observing the above rules by the candidates will entail cancellation of their registration.

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G-Type Wave Propagation in an Initially Stressed Fluid Saturated Viscoporoelastic Layer Lying over Heterogeneous Poroelastic Half Space

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Received September 6, 2020; revised March 1, 2021; accepted March 11, 2021

Abstract—G-type waves in an initially stressed poroelastic semi-infinite solid consists of a layer lying on a half space are investigated in the framework of Biot's theory of isotropic poroelasticity with some variants. Dispersion equation is obtained on solving the resultant Hill's differential equation with the aid of Laplace transform by the Valeev's method. Numerical examples for phase velocity, group velocity, angular frequency, and attenuation are presented graphically as a function of wavenumber for various values of viscosity, initial stress, and heterogeneity parameter. Some particular cases are discussed.

Keywords: G-type wave, viscoporoelastic layer, poroelastic half space, phase velocity, group velocity, angular frequency, attenuation, wavenumber

DOI: 10.3103/S0025654422010150

1. INTRODUCTION

Study of wave propagation in viscoporoelastic structures is useful in the domains such as Rock Mechanics, Geophysics, and Civil Engineering, particularly, in Structural Engineering. An earthquake generates two types of waves namely body waves and surface waves. Sato [1] studied surface waves, VI generation of Love, and other types of SH waves. The G type waves are horizontally polarized surface waves of shear type, named after Gutenberg [2, 3], who established the existence of a low velocity layer in the Earth mantle. The G-type waves propagate with the group velocity about 4.4 km/s [4–6]. Ewing *et al.* [7] documented the works on elastic waves in layered media. The G-type wave is an exceptional type of the Love wave, its duration is more, and speed is high when compared with that of the Love wave. Several researchers have investigated G-type waves, some works are cited here as follows. Bath studied shadow zones, travel times, and the energies of longitudinal seismic waves in presence of an asthenosphere low-velocity layer [8]. The G-type wave resulted in the earthquake in 1960 is studied with regard to absorption and velocity [9]. Generation and propagation of G-type waves of the Niigata earthquake in 1964 are discussed [10]. In the paper [10], earthquake moment, released energy, and stress-strain drop from the G-type wave spectrum are estimated. Notable works of G-type waves can be seen in the papers [11–14]. In these papers, propagation of G-type waves is investigated using the Valeev's method [15].Vardoulakis *et al.* [16] discussed SH waves in a homogeneous gradient-elastic half-space with surface energy. Propagation of Love waves in a transversely isotropic fluid-saturated porous layered half-spaces studied by Wang *et al.* [17]. It is observed that there are significant effects of gravity, porosity, and anisotropy on the wave characteristics [18]. Gupta *et al.* studied the propagation of Love waves in a non-homogeneous substratum over an initially stressed heterogeneous half-space [19]. The Love waves in fiber-reinforced layer over a porous half-space are investigated by Chattaraj and Samal [20]. Kundu *et al.* studied the propagation of G-type waves in a heterogeneous layer lying over an initial stressed half space [21]. In the paper [21], it is confirmed that an initial stress, and an inhomogeneity parameter have effect on the propagation. Xu *et al.* [22] discussed G-type dispersion equation under suppressed rigid boundary by analytic approach. In the paper [23], it is concluded that an initial stress and heterogeneity have significant effect on dispersion of G- type waves. The Earth is porous and magnetic in nature, because of mineral deposits in it. In this context, shear wave propagation in magneto poroelastic medium sandwiched between self-reinforced poroelastic medium and poroelastic half space is studied in the framework of Biot's theory of poroelasticity [24, 25]. From the paper [24], it is clear that heterogeneity parameter, inhomogeneity parameter, and reinforcement parameter have significant influence on the wave characteristics. In the above mentioned papers, authors have not considered viscosity, an initial stress, and heterogeneity all at a time which may not be realistic. Hence, in this paper, all the above are taken into the account in the framework of Biot's theory. Particular cases are investigated numerically. The results are compared with that of the earlier papers.

The rest of the paper is organized as follows. In Section 2, constitutive equations, formulation, and the solution of the problem are presented. In Section 3, boundary conditions and frequency equation are presented. Particular cases are discussed in Section 4. Numerical results are discussed in Section 5. Finally, conclusion is given in Section 6.

2. CONSTITUTIVE EQUATIONS, FORMULATION AND SOLUTION OF THE PROBLEM

Equations of motion for isotropic poroelastic medium in absence of dissipation are [25]

$$\begin{gathered} N\nabla^2 u + \nabla((A+N)e + Q\varepsilon) = \frac{\partial^2}{\partial t^2}(\rho_{11}u + \rho_{12}U), \\ \nabla(Qe + R\varepsilon) = \frac{\partial^2}{\partial t^2}(\rho_{12}u + \rho_{22}U). \end{gathered} \tag{2.1}$$

In Eq. (2.1), $\mathbf{u} = (u, v, w)$ and $\mathbf{U} = (U, V, W)$ are displacement vectors of solid and fluid, respectively, e and ε are the dilatations of solid and fluid respectively, $P = A + 2N$, Q, and R are all poroelastic constants. ρ_{11}, ρ_{12} and ρ_{22} are mass coefficients, and t is time. The solid stresses σ_{ij} and fluid pressure s are given by [25]

$$\sigma_{ij} = 2Ne_{ij} + (Ae + Q\varepsilon)\delta_{ij}, \quad (i, j = 1,2,3), \quad s = Qe + R\varepsilon. \tag{2.2}$$

In Eq. (2.2), δ_{ij} is well-known Kronecker delta function. Consider a viscoporoelastic layer of thickness H under initial stress P_1 (say) lying over heterogeneous poroelastic half space under initial stress P_2 (say). Both the layer and half space are semi-infinite as shown in the Fig. 1. This set up is quite possible in the structure of Earth. The interface is at $z = 0$, and the upper layer is bounded by the plane $z = -H$, the x-axis is parallel to the layer in the direction of propagation. The positive z-axis is oriented vertically downwards. From the figure, it is clear that upper viscoporoelastic layer and heterogeneous poroelastic half space occupy the region $(-H \le z < 0)$, $(0 \le z < \infty)$, respectively. In the following sub sections, the wave propagation is considered individually in the layer and half space.

2.1. Wave Propagation in Upper Viscous Isotropic Poroelastic Initially Stressed Layer

Consider horizontally polarized surface wave is of a shear type in the layer. Then the displacement components here are $u_1 = w_1 = 0$, $v_1 = v_1(x,z,t)$, $U_1 = W_1 = 0$, $V_1 = V_1(x,z,t)$. Because of variation in viscosity and presence of initial stress, the equations of motion (1) for the layer [23, 25] take the following form:

$$\begin{gathered} \left(N_1 + N_1'\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial z^2}\right) - \frac{P_1}{2}\frac{\partial^2 v_1}{\partial x^2} = \frac{\partial^2}{\partial t^2}(\rho_{11}v_1 + \rho_{12}V_1), \\ 0 = \frac{\partial^2}{\partial t^2}(\rho_{12}v_1 + \rho_{22}V_1). \end{gathered} \tag{2.3}$$

In Eq. (2.3), N_1 is the shear modulus of the solid, N_1' is the viscosity of fluid. Eq. (2.3) can also be written as

$$\left(N_1 + N_1'\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial z^2}\right) - \frac{p_1}{2}\frac{\partial^2 v_1}{\partial x^2} = d_1\frac{\partial^2 v_1}{\partial t^2}, \quad d_1 = \rho_{11} - \frac{\rho_{12}^2}{\rho_{22}}. \tag{2.4}$$

For the G-type wave, the displacement component $v_1(z)$ is assumed as

$$v_1(x,z,t) = v_1(z)e^{ik(x-ct)}. \tag{2.5}$$

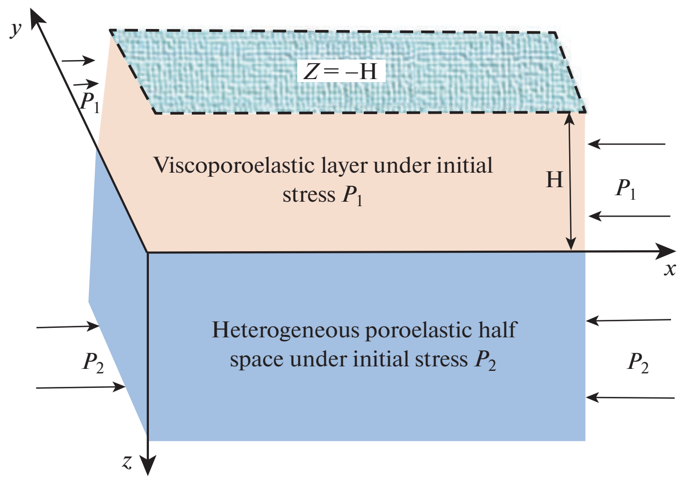

Fig. 1. Geometry of the problem.

In Eq. (2.5), k is complex wavenumber, c is phase velocity. Substituting Eq. (2.5) in Eq. (2.4), and solving the resultant differential equation, one obtains the following solution:

$$v_1(z) = c_1 \cos(\psi(z + c_2))e^{i(kx-\omega t)}, \tag{2.6}$$

where, $\psi = \left(\left(k_1^2 - k^2\left(1 - \frac{p_1}{2\overline{N}_1}\right)\right)\right)^{\frac{1}{2}}, k_1^2 = \frac{d_1\omega^2}{\overline{N}_1}, \overline{N_1} = N_1 + i\omega N_1^1, \omega = kc$, and c_1, c_2 are arbitrary constants.

2.2. Wave Propagation in Lower Heterogeneous Poroelastic Half Space under Initial Stress

For the same reason mentioned in the subsection 2.1, the displacement components here are $u_2 = w_2 = 0, v_2 = v_2(x,z,t)$, $U_2 = W_2 = 0, V_2 = V_2(x,z,t)$. Because of nature of the half space, there are the variants in stress terms, and consequently, the equations of motion take the form [23, 25]:

$$\frac{\partial}{\partial x}\left((N_2(1-\varepsilon\cos\alpha z)\frac{\partial v_2}{\partial x}\right) + \frac{\partial}{\partial z}\left((N_2(1-\varepsilon\cos\alpha z)\frac{\partial v_2}{\partial x}\right) - \frac{P_2}{2}\frac{\partial^2 v_2}{\partial x^2} = \frac{\partial^2}{\partial t^2}(\rho_{11}^*(1-\varepsilon\cos\alpha z)v_2)$$
$$+\frac{\partial^2}{\partial t^2}(\rho_{12}^*(1-\varepsilon\cos\alpha z)V_2),$$

$$0 = \frac{\partial^2}{\partial t^2}(\rho_{11}^*(1-\varepsilon\cos\alpha z)v_2 + \rho_{22}^*(1-\varepsilon\cos\alpha z)V_2). \tag{2.7}$$

In Eq. (2.7), because of heterogeneity with respect to $z$axis, shear modulus N_2 and densities, are, written as $N_2(1-\varepsilon\cos\alpha z)$, $\rho_{11}^*(1-\varepsilon\cos\alpha z)$, $\rho_{12}^*(1-\varepsilon\cos\alpha z)$, $\rho_{22}^*(1-\varepsilon\cos\alpha z)$, here ε is a small positive constant, α is real depth parameter, and $*$ stands for the quantities in the half space. Then, Eq. (2.7) can be written as

$$\frac{\partial}{\partial x}\left((N_2(1-\varepsilon\cos\alpha z)\frac{\partial v_2}{\partial x}\right) + \frac{\partial}{\partial z}\left((N_2(1-\varepsilon\cos\alpha z)\frac{\partial v_2}{\partial x}\right) - \frac{P_2}{2}\frac{\partial^2 v_2}{\partial x^2} = d_2(1-\varepsilon\cos\alpha z)\frac{\partial^2 v_2}{\partial t^2}. \tag{2.8}$$

In Eq. (2.8), $d_2 = \rho_{11}^* - \frac{\rho_{12}^{*2}}{\rho_{22}^*}$.

For the G-wave propagation, the displacement component v_2 is assumed as

$$v_2(x,z,t) = v_2(z)e^{ik(x-ct)}. \tag{2.9}$$

From Eqs. (2.8) and (2.9), the following differential equation is obtained:

$$\frac{d^2v_2(z)}{dz^2}+\left(\frac{d_2c^2}{N_2}+\left(\frac{P_2}{2N_2}-1\right)\right)k^2v_2(z)+e^{-i\alpha z}\left(\frac{-\varepsilon d_2}{2N_2}k^2c^2v_2(z)+\frac{\varepsilon}{2}k^2v_2(z)-\frac{\varepsilon}{2}\frac{d^2v_2(z)}{dz^2}+\frac{\varepsilon\alpha i}{2}\frac{dv_2(z)}{dz}\right)$$
$$+e^{i\alpha z}\left(\frac{-\varepsilon d_2}{2N_2}k^2c^2v_2(z)+\frac{\varepsilon}{2}k^2v_2(z)-\frac{\varepsilon}{2}\frac{d^2v_2(z)}{dz^2}-\frac{\varepsilon\alpha i}{2}\frac{dv_2(z)}{dz}\right)=0. \tag{2.10}$$

Equation (2.10) is well known form of the Hill's differential equation which can be solved by the Valeev's method [15] with the aid of Laplace transform with respect to the variable z. On multiplying Eq. (2.10) both sides by e^{-sz}, and then integrating with respect to z form 0 to ∞, one obtains

$$\int_0^\infty e^{-(s+i\alpha)z}\left(\frac{-\varepsilon d_2}{2N_2}k^2c^2v_2(z)+\frac{\varepsilon}{2}k^2v_2(z)-\frac{\varepsilon}{2}\frac{d^2v_2}{dz^2}+\frac{\varepsilon\alpha i}{2}\frac{dv_2}{dz}\right)dz$$
$$+\int_0^\infty e^{-(s-i\alpha)z}\left(\frac{-\varepsilon d_2}{2N_2}k^2c^2v_2(z)+\frac{\varepsilon}{2}k^2v_2(z)-\frac{\varepsilon}{2}\frac{d^2v_2}{dz^2}-\frac{\varepsilon\alpha i}{2}\frac{dv_2}{dz}\right)dz \tag{2.11}$$
$$+\int_0^\infty \frac{d^2v_2}{dz^2}+\left(\frac{d_2c^2}{N_2}+\left(\frac{P_2}{2N_2}-1\right)\right)k^2v_2(z)dz=0.$$

The Laplace Transform of $v_2(z)$ is $\int_0^\infty e^{-sz}v_2(z)dz=F(s)$ (Say). Applying Laplace transform to Eq. (2.11), one obtains

$$F(s+i\alpha)\left(\frac{-\varepsilon d_2}{2N_2}k^2c^2+\frac{\varepsilon}{2}k^2-\frac{\varepsilon}{2}(s+i\alpha)^2+\frac{\varepsilon\alpha i}{2}(s+i\alpha)\right)$$
$$+F(s-i\alpha)\left(\frac{-\varepsilon d_2}{2N_2}k^2c^2+\frac{\varepsilon}{2}k^2-\frac{\varepsilon}{2}(s-i\alpha)^2+\frac{\varepsilon\alpha i}{2}(s-i\alpha)\right) \tag{2.12}$$
$$+(s^2-r^2)F(s)=s\sigma_1+\sigma_2,$$

where $\sigma_1=(1-\varepsilon)v_2(0), \sigma_2=(1-\varepsilon)q(o)$, $q(o)=v_2'(o)$, and $r^2=\left(1-\frac{P_2}{2N_2}-\frac{d_2c^2}{N_2}\right)k^2$. The function $F(s)$ is determined as follows: Replacing s by $s+i\alpha m$, and then dividing throughout Eq. (2.12) by $(i\alpha m)^n$, $(m\neq 0)$, the following system of linear algebraic equations in the quantities $F(s+i\alpha m)$, $m=0,\pm1,\pm2,\ldots$, is obtained:

$$(i\alpha m)^{-n}F(s+i\alpha(m+1))\left(\frac{-\varepsilon d_2}{2N_2}k^2c^2+\frac{\varepsilon}{2}k^2-\frac{\varepsilon}{2}(s+i\alpha(m+1)^2)+\frac{\varepsilon\alpha i}{2}(s+i\alpha(m+1))\right)$$
$$+(i\alpha m)^{-n}F(s-i\alpha(m-1))\left(\frac{-\varepsilon d_2}{2N_2}k^2c^2+\frac{\varepsilon}{2}k^2-\frac{\varepsilon}{2}(s-i\alpha(m-1)^2)+\frac{\varepsilon\alpha i}{2}(s-i\alpha(m-1))\right) \tag{2.13}$$
$$+(i\alpha m)^{-n}((s+i\alpha m)^2-r^2)F(s+i\alpha m)=(i\alpha m)^{-n}(\sigma_1(s+i\alpha m)+\sigma_2),$$

for $m=0$, Eq. (2.13) is reduced to Eq. (2.12) on assuming $(i\alpha m)^{-n}=1$. Solving Eq. (2.13), one obtains $F(s)$ as the ratio of two determinants, and is given by $F(s)=\frac{\Delta_3}{\Delta_4}$, where

$$\Delta_3=\begin{vmatrix} (i\alpha)^{-n}((s+i\alpha)^2-r^2) & (i\alpha)^{-n}((s+i\alpha)\sigma_1+\sigma_2) & 0 \\ \begin{matrix}\frac{-\varepsilon d_2}{2N_2}k^2c^2+\frac{\varepsilon k^2}{2}+\\ \frac{\varepsilon\alpha i}{2}(s+i\alpha)-\frac{\varepsilon}{2}(s+i\alpha)^2\end{matrix} & s\sigma_1+\sigma_2 & \begin{matrix}\frac{-\varepsilon d_2}{2N_2}k^2c^2+\frac{\varepsilon k^2}{2}-\\ \frac{\varepsilon\alpha i}{2}(s-i\alpha)-\frac{\varepsilon}{2}(s-i\alpha)^2\end{matrix} \\ 0 & (-i\alpha)^{-n}((s-i\alpha)\sigma_1+\sigma_2) & (-i\alpha)^{-n}((s-i\alpha)^2-r^2) \end{vmatrix},$$

$$\Delta_4 = \begin{vmatrix} (i\alpha)^{-n}((s+i\alpha)^2 - r^2) & (-i\alpha)^{-n}\left(\frac{-\varepsilon d_2}{2N_2}k^2c^2 + \frac{\varepsilon k^2}{2} - \frac{\varepsilon\alpha i s}{2} - \frac{\varepsilon s^2}{2}\right) & 0 \\ \frac{-\varepsilon d_2}{2N_2}k^2c^2 + \frac{\varepsilon k^2}{2} + \frac{\varepsilon\alpha i}{2}(s+i\alpha) - \frac{\varepsilon}{2}(s+i\alpha)^2 & s^2 - r^2 & \frac{-\varepsilon d_2}{2N_2}k^2c^2 + \frac{\varepsilon k^2}{2} - \frac{\varepsilon\alpha i}{2}(s-i\alpha) - \frac{\varepsilon}{2}(s-i\alpha)^2 \\ 0 & (-i\alpha)^{-n}\left(\frac{-\varepsilon d_2}{2N_2}k^2c^2 + \frac{\varepsilon k^2}{2} + \frac{\varepsilon\alpha i s}{2} - \frac{\varepsilon s^2}{2}\right) & (-i\alpha)^{-n}((s-i\alpha)^2 - r^2) \end{vmatrix}.$$

Neglecting ε^2 and higher order terms in Δ_3 as ε is very small, one obtains

$$\alpha^{2n}\Delta_3 = (s\sigma_1 + \sigma_2)((s+i\alpha)^2 - r^2)((s-i\alpha)^2 - r^2) + (\sigma_1(s-i\alpha) + \sigma_2)((s+i\alpha)^2 - r^2),$$
$$\left(\frac{\varepsilon d_2}{2N_2}k^2c^2 - \frac{\varepsilon k^2}{2} + \frac{\varepsilon\alpha i}{2}(s-i\alpha) + \frac{\varepsilon}{2}(s-i\alpha)^2\right) + (\sigma_1(s+i\alpha) + \sigma_2)((s-i\alpha)^2 - r^2),$$
$$\left(\frac{\varepsilon d_2}{2N_2}k^2c^2 - \frac{\varepsilon k^2}{2} - \frac{\varepsilon\alpha i}{2}(s+i\alpha) + \frac{\varepsilon}{2}(s+i\alpha)^2\right).$$

Neglecting ε^2 and higher order terms in Δ_4, one obtains

$$\alpha^{2n}\Delta_4 = ((s+i\alpha)^2 - r^2)((s-i\alpha)^2 - r^2)(s^2 - r^2).$$

Then

$$F(s) = \frac{s\sigma_1 + \sigma_2}{s^2 - r^2} + \frac{\varepsilon}{2}\frac{v_2(0)(s+i\alpha) + q(0)}{(s^2 - r^2)((s+i\alpha)^2 - r^2)}\left(\frac{d_2 k}{N_2} - k^2 - \alpha i(s+i\alpha) + (s+i\alpha)^2\right)$$
$$+ \frac{\varepsilon}{2}\frac{v_2(0)(s-i\alpha) + q(0)}{(s^2 - r^2)((s-i\alpha)^2 - r^2)}\left(\frac{d_2 k}{N_2} - k^2 + \alpha i(s-i\alpha) + (s-i\alpha)^2\right).$$

The factor $v_2(z)$ is given by

$$v_2(z) = \frac{1}{2\pi i}\int_{\alpha - i\infty}^{\alpha + i\infty} F(s)e^{sz}ds = R_1 + R_2 + R_3, \tag{2.14}$$

where R_1, R_2 and R_3 are the residues at the poles $s = r, s = r + i\alpha, s = r - i\alpha$, respectively. The residue R_1 at the pole $s = r$ is given by

$$R_1 = \left(\frac{rv_2(0) + q(0)}{2r}\right)\left((1-\varepsilon) + \frac{\varepsilon r^2}{\alpha^2 + r^2}\right)e^{rz} - \frac{\varepsilon L}{\alpha^2 + 4r^2}\left(\frac{q(0) - rv_2(0)}{2r}\right)e^{rz} + \frac{\varepsilon v_2(0)}{2}\frac{\alpha^2 + 2r^2}{\alpha^2 + 4r^2}e^{rz}. \tag{2.15}$$

Similarly, the residues at $s = r + i\alpha, s = r - i\alpha$ are respectively, given by

$$R_2 = \frac{-i\varepsilon}{4\alpha}\left(\frac{(rv_2(0) + q(0))(L + r^2 + i\alpha r)}{r(2r + i\alpha)}\right)e^{(r+i\alpha)z}, \tag{2.16}$$

$$R_3 = \frac{i\varepsilon}{4\alpha}\left(\frac{(rv_2(0) + q(0))(L + r^2 + i\alpha r)}{r(2r - i\alpha)}\right)e^{(r-i\alpha)z}, \tag{2.17}$$

where $L = \dfrac{d_2 k^2 c^2}{N_2} - k^{2.}$.

From the Eqs. (2.15)–(2.17), it is clear that the conditions for large amount of energy near the surface are [13]

$$rv_2(0) + q(0) = 0, \tag{2.18}$$

$$rv_2(0) - q(0) = 0, \tag{2.19}$$

$$2r^2 + \alpha^2 = 0. \tag{2.20}$$

3. BOUNDARY CONDITIONS AND FREQUENCY EQUATION

The boundary conditions are assumed as follows:

(i) Displacements are continuous at the interface $z = 0$, that is $v_1 = v_2$.

(ii) Stresses are continuous at $z = 0$, i.e. $\overline{N_1}\dfrac{\partial v_1}{\partial z} = N_2(1-\varepsilon\cos\alpha z)\dfrac{\partial v_2}{\partial z}$. (3.1)

(iii) The upper surface is stress free i.e. at $z = -H$, that is $\dfrac{\partial v_1}{\partial z} = 0$.

From the energy conditions Eqs. (2.18), (2.19), and boundary conditions Eq. (3.1), one obtains

$$\tan\left(\left\{\frac{c^2}{\overline{\beta_1^2}} + \frac{P_1}{2\overline{N_1}} - 1\right\}^{\frac{1}{2}} kH\right) = \pm\frac{N_2(1-\varepsilon)\left(1 - \dfrac{P_2}{2N_2} - \dfrac{c^2}{\beta_2^2}\right)^{\frac{1}{2}}}{\overline{N_1}\left(\dfrac{c^2}{\overline{\beta_1^2}} + \dfrac{P_1}{2\overline{N_1}} - 1\right)^{\frac{1}{2}}}, \tag{3.2}$$

where $\overline{\beta_1} = \sqrt{\dfrac{\overline{N_1}}{d_1}}$ is the shear wave velocity for upper layer, and $\beta_2 = \sqrt{\dfrac{N_2}{d_2}}$ is the shear wave velocity for lower half space. Equating real parts on both sides of Eq. (3.2), one obtains

$$\frac{\tan(xkH)(1-(\tanh(ykH))^2)}{1+(\tan(xkH)\tanh(ykH))^2} = \frac{N_2(1-\varepsilon)\left(1 - \dfrac{P_2}{2N_2} - \dfrac{c^2}{\beta_2^2}\right)^{\frac{1}{2}}(xN_1 + y\omega N_1')}{(xN_1 + y\omega N_1')^2 + (x\omega N_1' - yN_1)^2}. \tag{3.3}$$

In Eq. (3.3),

$$x = \sqrt{\left(\frac{A_1 \pm \sqrt{A_1^2 + A_2^2}}{2}\right)}, \quad y = \frac{A_2}{2x},$$

$$A_1 = \frac{(2d_1c^2 + P_1)}{2N_1\left(1 + \left(\dfrac{\omega N_1'}{N_1}\right)^2\right)} - 1, \quad A_2 = \frac{\dfrac{(2d_1c^2 + P_1)}{N_1}\dfrac{\omega N_1'}{N_1}}{2\left(1 + \left(\dfrac{\omega N_1'}{N_1}\right)^2\right)}$$

Equation (3.3) is the dispersion equation for G-type wave propagation in viscoporoelastic layer under initial stress lying over heterogeneous poroelastic half space under initial stress. Similarly, comparing imaginary parts on both the sides of Eq. (3.2), one obtains

$$\frac{(\tan^2 xkH + 1)\tanh ykH}{1+(\tan xkH\tanh ykH)^2} = \frac{N_2(1-\varepsilon)\left(1 - \dfrac{P_2}{2N_2} - \dfrac{c^2}{\beta_2^2}\right)^{\frac{1}{2}}(x\omega N_1' - yN_1)}{(xN_1 + y\omega N_1')^2 + (x\omega N_1' - yN_1)^2}. \tag{3.4}$$

From Eq. (2.20), one obtains

$$kc = \sqrt{\frac{N_2}{2d_2}\left(\left(1 - \frac{P_2}{2N_2}\right)2k^2\right) + \alpha^2}. \tag{3.5}$$

The group velocity $$U = \frac{d}{dk}(kc) = \frac{\beta_2\sqrt{2}k\left(1 - \dfrac{P_2}{2N_2}\right)}{\sqrt{2k^2\left(1 - \dfrac{P_2}{2N_2}\right) + \alpha^2}}. \tag{3.6}$$

4. PARTICULAR CASES

Case (i). Under the conditions $P_1 = 0, P_2 = P, \rho_{12} = 0, \rho_{22} = 0, \rho_{11}^* = 0, \rho_{22}^* = 0, d_1 = \rho_{11} = \rho_1, d_2 = \rho_{11}^* = \rho_2$, Eq. (3.3) represents the dispersion equation for the propagation of G-type wave in a viscoelastic layer lying over a heterogeneous elastic half space under an initial stress [23].

Case (ii). When $N_1' = 0$, eq. (23) reduces to

$$\tan\left(\left\{\frac{c^2}{\beta_1^2} + \frac{P_1}{2N_1} - 1\right\}^{\frac{1}{2}} kH\right) = \frac{N_2(1-\varepsilon)}{N_1} \frac{\left(1 - \frac{P_2}{2N_2} - \frac{c^2}{\beta_2^2}\right)^{\frac{1}{2}}}{\left(\frac{c^2}{\beta_1^2} + \frac{P_1}{2N_1} - 1\right)^{\frac{1}{2}}}. \tag{4.1}$$

In Eq. (4.1), $\beta_1 = \sqrt{\frac{N_1}{d_1}}, \beta_2 = \sqrt{\frac{N_2}{d_2}}$.

Equation (4.1) represents the dispersion equation for the G-type wave in an initially stressed poroelastic layer lying over a heterogeneous poroelastic half space, both are non-viscous.

Case (iii). When $N_1' = 0, \rho_{12} = 0, \rho_{22} = 0, \rho_{11}^* = 0, \rho_{22}^* = 0, d_1 = \rho_{11} = \rho_1, d_2 = \rho_{11}^* = \rho_2$, Eq. (3.3) reduces to

$$\tan\left(\left\{\frac{\rho_1 c^2}{N_1} + \frac{P_1}{2N_1} - 1\right\}^{\frac{1}{2}} kH\right) = \frac{N_2(1-\varepsilon)}{N_1} \frac{\left(1 - \frac{P_2}{2N_2} - \frac{\rho_2 c^2}{N_2}\right)^{\frac{1}{2}}}{\left(\frac{\rho_1 c^2}{N_1} + \frac{P_1}{2N_1} - 1\right)^{\frac{1}{2}}}. \tag{4.2}$$

Equation (4.2) represents the dispersion equation for the propagation of the G-type wave in an initially stressed elastic layer lying over a heterogeneous elastic half space, both are non-viscous.

Case (iv). Under the conditions of case (iii), and when $P_1 = 0, P_2 = 0, \varepsilon = 0$, Eq. (4.2) reduces to

$$\tan\left(\left\{\frac{c^2}{\beta_1^2} - 1\right\}^{\frac{1}{2}} kH\right) = \frac{N_2}{N_1} \frac{\left(1 - \frac{c^2}{\beta_2^2}\right)^{\frac{1}{2}}}{\left(\frac{c^2}{\beta_1^2} - 1\right)^{\frac{1}{2}}}. \tag{4.3}$$

Equation (4.3) represents the dispersion equation of Love waves in a homogeneous layer over a homogeneous half-space with $\beta_1 < c < \beta_2$, where $\beta_1 = \sqrt{\frac{N}{\rho_1}}, \beta_2 = \sqrt{\frac{N}{\rho_2}}$ [7].

5. NUMERICAL RESULTS

For numerical work, the sandstone layer saturated with water (say Mat-I) [26] lying over the sandstone half space saturated with kerosene (say Mat-II) [27] is considered. This kind of solid composition physically exists at the sea shore consists of oil reserves. The parameter values of these two poroelastic solids are given in the Table 1.

Employing these values in the frequency equations, the implicit relation between the wave characteristics is obtained. The wave characteristics phase velocity, angular frequency, group velocity, and attenuation coefficient are computed against wavenumber. The attenuation (Q^{-1}) is computed by using the following Eq. [28].

$$Q^{-1} = \frac{2\ \ (\textit{frequency of imaginary part in eq.}(3.2))}{\textit{frequency of real part in eq.}(3.2)}.$$

Table 1. Poroelastic solid parameters

Mat-I	N_1'	N_1	ρ_{11}	ρ_{12}	ρ_{22}
	2.8×10^{-12}	2.76×10^{-10}	18.6620×10^{-8}	0	22.1630×10^{-9}
Mat-II	N_2'	N_2	ρ_{11}^*	ρ_{12}^*	ρ_{22}^*
	0	9.22×10^{-10}	18.8876×10^{-8}	-19.6133×10^{-11}	21.0842×10^{-9}

The quantity $\frac{\omega N_1'}{N_1}$ is taken as 2.02 arbitrarily. The values are computed using bisection method implemented in MATLAB, and the results are depicted in Figs. 2–6. Figure 2a depicts the variation of phase velocity $\frac{c}{\beta_1}$ against wavenumber kH. Curves are plotted for different values of heterogeneous parameter ε, and for fixed values of initial stresses $P_1/2N_1$ and $P_2/2N_2$. From this figure, it is clear that as wavenumber increases phase velocity decreases. As heterogeneous parameter increases, phase velocity, in general, decreases. Also, comparison is made between the viscoporoelastic solid and the pertinent non viscous elastic solid. In this case, phase velocity decreases in the case of elastic solid, whereas, the trend is reversed in poroelastic case. Moreover, as wavenumber increases, phase velocity values are higher in viscoporoelastic case than that of elastic case. Figure 2b depicts the variation of phase velocity $\frac{c}{\beta_1}$ against the wavenumber kH. Curves are plotted for different values of initial stresses $P_1/2N_1$, $P_2/2N_2$ and for fixed values of heterogeneous parameter ε.

From Fig. 2b, it is observed that as wavenumber increases, phase velocity increases. As initial stress increases, phase velocity, in general, decreases. Figure 3 displays the influence of initial stress $P_2/2N_2$ on phase velocity for different values of ε in absence of initial stress.

From Fig. 3, it is clear that phase velocity decreases with increase of heterogeneous parameter ε as in the paper [23].

Figure 4a is pertaining to the results of angular frequency against wavenumber. Curves are plotted for different values of heterogeneous parameter ε, and for fixed initial stress. From this figure, it is clear that as wavenumber increases angular frequency diminishes, and increases with heterogeneity. Figure 4b depicts the variation of angular frequency against wavenumber for different values of initial stresses, and for fixed heterogeneous parameter ε.

From Fig. 4b, it is clear that angular frequency diminishes as wavenumber increases, and increases as initial stress increases. Figure 5 shows variation of group velocity against wavenumber for different values of depth parameter α.

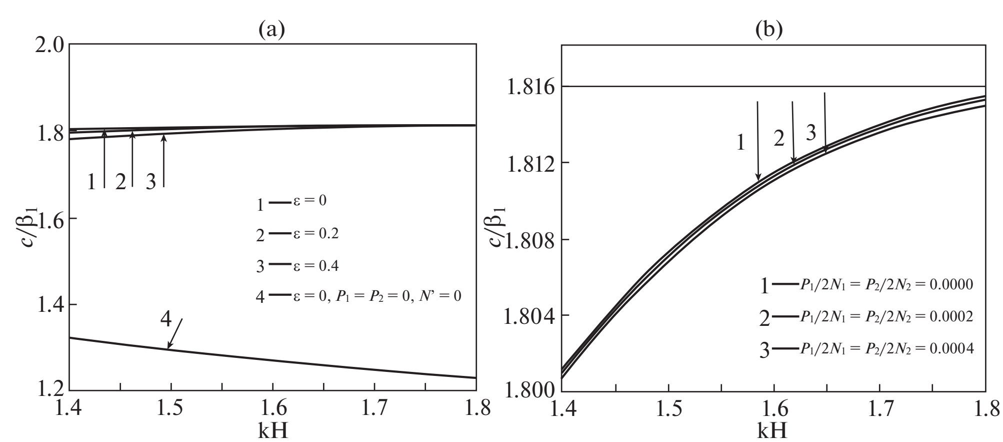

Fig. 2. (a) Variation of the phase velocity against wavenumber kH in viscoporoelastic case and elastic case. (b) Variation of the phase velocity against wavenumber kH for different values of $P_1/2N_1$ and $P_2/2N_2$.

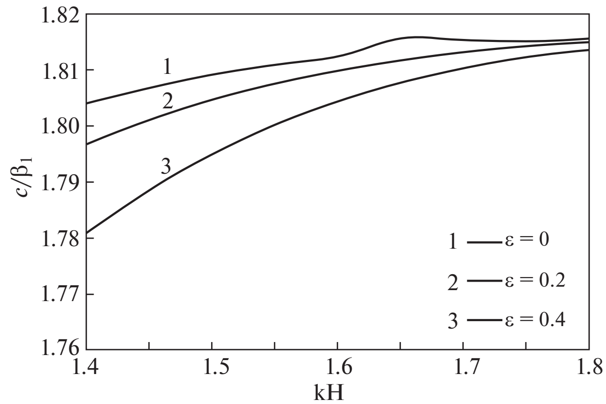

Fig. 3. Variation of the phase velocity against wavenumber kH for different values of heterogeneous parameter in absence of initial stress P_1 (layer and half space are elastic i.e. Ref [23]).

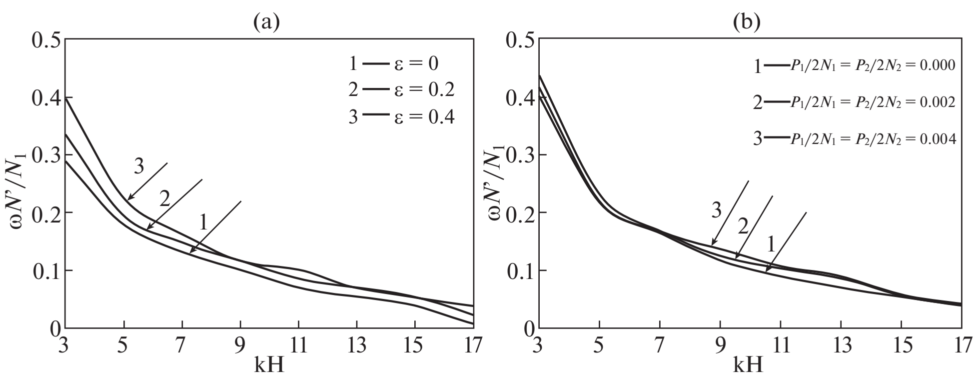

Fig. 4. (a) Variation of the angular frequency against wavenumber kH for different values of heterogeneous parameter ε. (b) Variation of the angular frequency against wavenumber kH for different values of initial stresses $P_1/2N_1$ and $P_2/2N_2$.

From Fig. 5, it is clear that group velocity increases with wavenumber, and decreases with depth parameter which is a natural phenomenon for surface waves. Finally, in Fig. 6, curves are plotted for attenuation coefficient as a function of wavenumber.

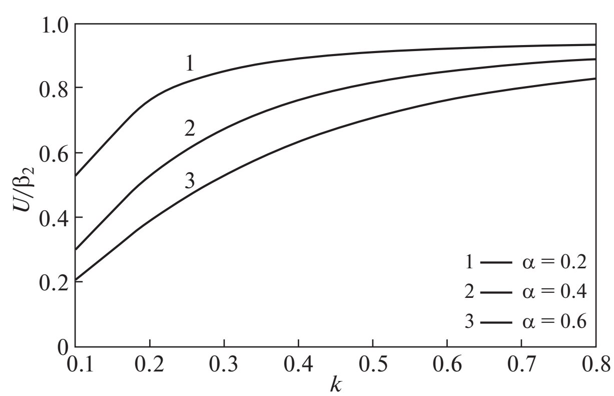

Fig. 5. Variation of the group velocity against wavenumber for different values of the depth parameter α.

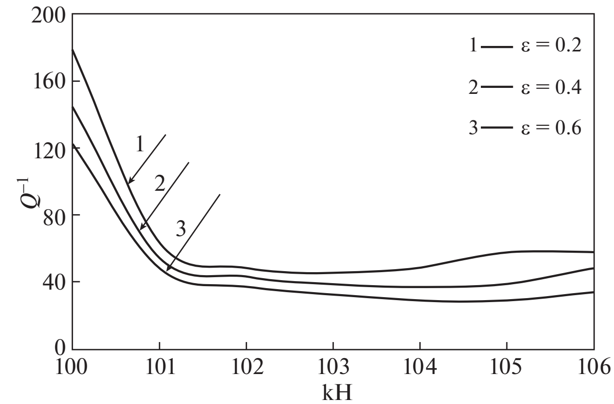

Fig. 6. Variation of the attenuation coefficient against wavenumber for different values of heterogeneous parameter ε.

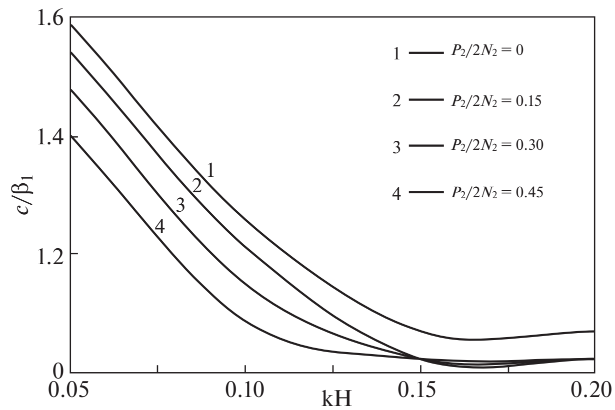

Fig. 7. Variation of phase velocity c/β_1 against wave number kH for different values of initial stress $P_2/2N_2$.

From Fig. 6, it is clear that as attenuation coefficient decreases with increase of wavenumber and heterogeneous parameter. For the validation purpose, the following substitutions are made as in [23] $\frac{d_2}{d_1} = 0.01, \frac{N_2}{N_1} = 0.1, \frac{\omega N_1^{'}}{N_1} = \frac{4\pi}{9}, P_1 = 0$, and the pertinent results are incorporated in Fig. 7.

From Fig. 7, it is seen that the phase velocity c/β_1 decreases with increase of wave number kH and the initial stress $P_2/2N_2$. All the results are in agreement and the trend is similar to that of Fig. 4 of [23].

6. CONCLUSIONS

The G-type wave propagation in an initially stressed viscoporoelastic layer lying over a heterogeneous initially stressed poroelastic half space is investigated. The solution in the case of upper layer is obtained in a usual way, whereas, for lower heterogeneous poroelastic half space, the resultant Hill's differential equation is solved by using Valeev's method with the aid of Laplace transform technique. From the numerical results, it is clear that as wavenumber increases, both phase velocity and group velocity increase, whereas, angular frequency and attenuation decrease for all the cases. It is seen that initial stresses, heterogeneous parameter, and depth parameter have influence on the wave characteristics. Above studies are useful for investigating the seismic waves of long period during earthquakes.

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STUDY OF SH-WAVE PROPAGATION IN AN INITIALLY STRESSED TRICLINIC LAYER SANDWICHED BETWEEN TRANSVERSELY ISOTROPIC ELASTIC AND HETEROGENEOUS POROELASTIC HALF-SPACES

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UDC 534.2

Abstract: In this paper, SH-wave propagation in an initially stressed triclinic layer welded between two half-spaces is investigated. The upper half-space is considered to be transversely isotropic and elastic, while the lower one is heterogeneous, isotropic, and poroelastic. In the case of the lower half-space, the problem is reduced to the Whittaker differential equation. Frequency equations are derived for the layer as a whole and half-spaces. It is found that the phase velocity is strongly influenced by the initial stress, porosity, and heterogeneity.

Keywords: SH-wave, triclinic layer, transversely isotropic half-space, inhomogeneity, poroelastic half-space, initial stress, porosity, phase velocity.

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INTRODUCTION

The Earth's interior is not homogeneous and consists of layers of different types of material properties due to different thermal conditions. Most of the information on the interior of the Earth and the cause of earthquakes are obtained from studies of seismic body waves of dilatational and shear type (P-waves and SH-waves, respectively). Dilatational waves cause a change in the volume of the material elements in the body, while shear waves produce a change in the shape of the material elements. The study of the horizontally polarized SH-wave is of considerable interest in the field of seismology, civil engineering, rock mechanics, and geophysics. SH-wave dispersion in elastic solids was studied by many researchers. Ewing et al. [1] documented the works on propagation of seismic waves. Bhattacharya [2] obtained an exact solution of the SH-wave equation for inhomogeneous media. Keith and Crampin [3] discussed seismic body waves in anisotropic layers. Payton [4] studied elastic wave propagation in transversely isotropic media. Notable progress in this domain was achieved in [5–9], where dispersion, reflection, and refraction of waves in triclinic and monoclinic crustal layers in the presence of irregularity, viscosity, and corrugated boundary were discussed. The Fourier transform technique, finite difference method, and Green's function approach were employed to study SH-wave propagation [10–12]. In [13–15], the authors discussed the effects of heterogeneity, viscosity, and sandy parameter on the process of SH-wave propagation. Parvez et al. [16] studied SH-wave dispersion in an irregular magneto-elastic anisotropic crustal layer over an irregular heterogeneous half-space. SH-wave propagation in a sandwiched medium between two elastic half-spaces and two anisotropic layers in the presence of a corrugated boundary, viscosity, material heterogeneity, and gravity was studied in [17, 18]. Scattering and

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dynamics of seismic waves in a triclinic medium were studied in [19–20]. On the other hand, SH-wave propagation in poroelastic solids in the framework of Biot's theory [21, 22] was investigated by many researchers. Chattopadhyay et al. [23] studied SH-waves in a porous layer of nonuniform thickness. Sahu et al. [24] discussed SH-wave propagation in a poroelastic sandwiched medium between two elastic half-spaces. Balu et al. [25] studied shear wave propagation in a magneto-poroelastic dissipative isotropic medium sandwiched between two poroelastic half-spaces. Recently, Sindhuja et al. [26] studied shear wave propagation in a magneto-poroelastic medium sandwiched between a self-reinforced poroelastic medium and a poroelastic half-space.

Beneath the Earth's surface, porous layers are naturally abundant. In general, the pores contain hydrocarbon deposits such as gas and oil. The body wave behaviors in anisotropic and isotropic media are fundamentally different. The study of SH-waves in a triclinic medium between semi-infinite transversely isotropic and poroelastic half-spaces is important for geophysical problems, and its results can be used to solve problems in exploration of hydrocarbons, mining crystals, and metals inside the Earth.

Motivated by the aforementioned facts, we studied SH-wave propagation in a triclinic medium sandwiched between a transversely isotropic half-space and an initially stressed inhomogeneous poroelastic half-space.

1. FORMULATION AND SOLUTION OF THE PROBLEM

Consider the propagation of an SH-wave in a triclinic layer under an initial stress P_1 welded between two half-spaces. The upper one is transversely isotropic and elastic, while the lower half-space is heterogeneous, poroelastic, and subjected to an initial stress P_2 (Fig. 1). This configuration is fairly possible in the Earth's structure. A three-dimensional coordinate system with the origin at the interface of the triclinic layer and the lower half-space is considered. The x axis is parallel to the layer in the direction of wave propagation, and the z axis is oriented vertically downwards. The thickness of the triclinic layer is H $(-H < z < 0)$. The z coordinate varies in the interval $-\infty < z < -H$ in the upper half-space and in the interval $z > 0$ in the lower porous half-space.

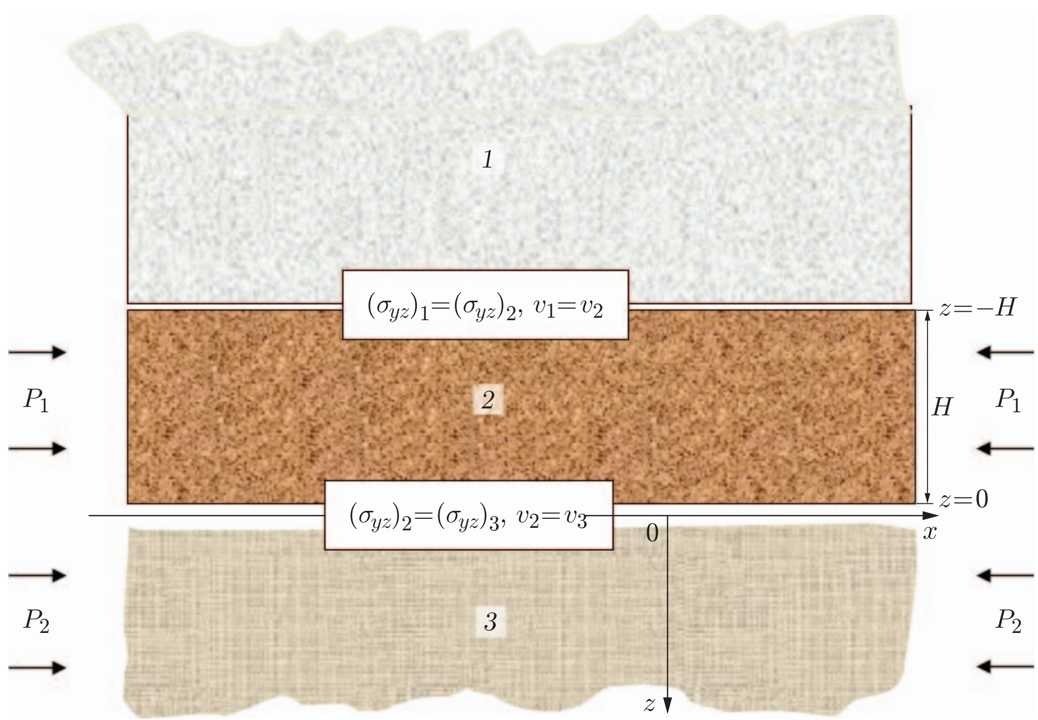

Fig. 1. Geometry of the problem: (*1*) transversely isotropic half-space; (*2*) triclinic layer; (*3*) heterogeneous poroelastic half-space.

1.1. *Wave Propagation in the Transversely Isotropic Elastic Upper Half-Space* ($-\infty < z < -H$)

Consider a horizontally polarized shear wave in the upper half-space propagating in the x direction with displacement in the y direction. Then the displacement components are

$$u_1 = w_1 = 0, \qquad v_1 = v_1(x, z, t).$$

The well-known wave equation has the form [22]

$$N_1 \frac{\partial^2 v_1}{\partial x^2} + L_1 \frac{\partial^2 v_1}{\partial z^2} = \rho_1 \frac{\partial^2 v_1}{\partial t^2}, \tag{1.1}$$

where N_1 and L_1 are the rigidities in the x and z directions, respectively, and ρ_1 is the density parameter.

For the SH-type wave, the displacement component v_1 can be written as

$$v_1(x, z, t) = f_1(z)\, \mathrm{e}^{i(kx - wt)}, \tag{1.2}$$

where k is the wavenumber, $w = ck$ is the angular frequency, c is the phase velocity, and $f_1(z)$ is the amplitude. Substituting Eq. (1.2) into Eq. (1.1) and solving the resultant differential equation, one obtains the amplitude

$$f_1(z) = A_1\, \mathrm{e}^{S_1 z} + A_2\, \mathrm{e}^{-S_1 z}$$

($S_1 = \sqrt{N_1/L_1 - c^2/\beta_1^2}\, k$, $\beta_1 = \sqrt{L_1/\rho_1}$ is the shear wave velocity in the upper half-space, and A_1 and A_2 are arbitrary constants). As the condition $f_1(z) \to 0$ has to be satisfied as $z \to -\infty$, then

$$v_1(x, z, t) = A_1\, \mathrm{e}^{S_1 z}\, \mathrm{e}^{i(kx - wt)}\,. \tag{1.3}$$

1.2. *Wave Propagation in the Triclinic Layer* ($-H < z < 0$)

Let u_2, v_2, and w_2 be the displacement vector components in the triclinic layer in the x, y, and z directions, respectively. The stress-strain relations for the anisotropic triclinic layer are written as

$$\sigma_i = \sum_{j=1}^{6} C_{ij} e_j, \qquad i = 1, 2, \ldots, 6,$$

where σ_i and e_i $(i = 1, 2, \ldots, 6)$ are the stress and strain components,

$$\begin{gathered} C_{ij} = C_{ji}, \quad \sigma_1 = \sigma_{xx}, \quad \sigma_2 = \sigma_{yy}, \quad \sigma_3 = \sigma_{zz}, \quad \sigma_4 = \sigma_{yz} = \sigma_{zy}, \\ \sigma_5 = \sigma_{xz} = \sigma_{zx}, \quad \sigma_6 = \sigma_{xy} = \sigma_{yx}, \\ e_1 = e_{xx}, \quad e_2 = e_{yy}, \quad e_3 = e_{zz}, \quad e_4 = e_{yz} = e_{zy}, \quad e_5 = e_{xz} = e_{zx}, \quad e_6 = e_{xy} = e_{yx}, \\ 2e_{xy} = \frac{\partial u_2}{\partial y} + \frac{\partial v_2}{\partial x}, \qquad 2e_{yz} = \frac{\partial v_2}{\partial z} + \frac{\partial w_2}{\partial y}, \qquad 2e_{zx} = \frac{\partial w_2}{\partial x} + \frac{\partial u_2}{\partial y}. \end{gathered} \tag{1.4}$$

For SH-waves propagating in the x direction with displacement in the y direction, the expressions for the displacement vector components are

$$u_2 = w_2 = 0, \qquad v_2 = v_2(x, z, t). \tag{1.5}$$

It follows from Eqs. (1.4) and (1.5) that

$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma_5 = 0.$$

The equation of motion in the triclinic layer under the initial stress P_1 in the absence of body forces has the following form [2]:

$$\frac{\partial\, (\sigma_6)}{\partial x} + \frac{\partial\, (\sigma_4)}{\partial z} - \frac{P_1}{2} \frac{\partial^2 v_2}{\partial x^2} = \rho_2 \frac{\partial^2 v_2}{\partial t^2}. \tag{1.6}$$

It follows from Eqs. (1.4) and (1.6) that

$$\Big(C_{66} - \frac{P_1}{2}\Big)\frac{\partial^2 v_2}{\partial x^2} + 2C_{46}\,\frac{\partial^2 v_2}{\partial x\,\partial z} + C_{44}\,\frac{\partial^2 v_2}{\partial z^2} = \rho_2\,\frac{\partial^2 v_2}{\partial t^2}, \tag{1.7}$$

where ρ_2 is the density of the triclinic layer material, and C_{44}, C_{46}, and C_{66} are the elastic constants of the triclinic layer material.

The solution of Eq. (1.7) can be written as

$$v_2(x,z,t) = f_2(z)\,\mathrm{e}^{i(kx-wt)}\,. \tag{1.8}$$

Substituting Eq. (1.8) into Eq. (1.7), one obtains

$$f_2(z) = \mathrm{e}^{-az/2}\,(A_3\cos qz + A_4\sin qz).$$

Thus, the displacement of the SH-wave in the triclinic layer is

$$v_2(x,z,t) = \mathrm{e}^{-az/2}\,(A_3\cos qz + A_4\sin qz)\,\mathrm{e}^{i(kx-wt)}, \tag{1.9}$$

where $q = \sqrt{b^2 - a^2/4}$, $a = 2ikC_{46}/C_{44}$, $b = \sqrt{P_1/(2C_{44}) - C_{66}/C_{44} + c^2/\beta_2^2}\,k$, $\beta_2 = \sqrt{C_{44}/\rho_2}$ is the shear wave velocity in the triclinic layer, and A_3 and A_4 are arbitrary constants.

1.3. Wave Propagation in the Lower Heterogeneous Isotropic Poroelastic Half-Space ($z > 0$)

Let u_3, v_3, w_3 and U_3, V_3, W_3 be the components of the displacement vector in the solid and fluid, respectively. Then the components of the displacement vector in the y direction in the SH-wave propagating in the x direction can be written as

$$u_3 = w_3 = 0, \quad v_3 = v_3(x,z,t), \quad U_3 = W_3 = 0, \quad V_3 = V_3(x,z,t).$$

The equations of motion for the lower poroelastic half-space under the initial stress P_2 have the following form [21, 22]:

$$\begin{gathered}\frac{\partial}{\partial x}\Big(N_3^*\,\frac{\partial v_3}{\partial x}\Big) + \frac{\partial}{\partial z}\Big(N_3^*\,\frac{\partial v_3}{\partial z}\Big) - \frac{P_2^*}{2}\,\frac{\partial^2 v_3}{\partial x^2} = \rho_{11}^*\,\frac{\partial^2 v_3}{\partial t^2} + \rho_{12}^*\,\frac{\partial^2 V_3}{\partial t^2},\\ \rho_{12}^*\,\frac{\partial^2 v_3}{\partial t^2} + \rho_{22}^*\,\frac{\partial^2 V_3}{\partial t^2} = 0,\end{gathered} \tag{1.10}$$

In these equations,

$$\begin{gathered}N_3^* = N_3(1+\alpha z), \qquad \rho_{11}^* = \rho_{11}(1+\alpha z), \qquad \rho_{12}^* = \rho_{12}(1+\alpha z),\\ \rho_{22}^* = \rho_{22}(1+\alpha z), \qquad P_2^* = P_2(1+\alpha z),\end{gathered} \tag{1.11}$$

N_3 is the shear modulus, ρ_{11}, ρ_{12}, and ρ_{22} are the mass coefficients, and α is the constant with the dimension of inverse length.

It follows from Eqs. (1.10) and (1.11) that

$$N_3(1+\alpha z)\,\frac{\partial^2 v_3}{\partial x^2} + N_3\alpha\,\frac{\partial v_3}{\partial z} + N_3(1+\alpha z)\,\frac{\partial^2 v_3}{\partial z^2} - \frac{P_2}{2}\,(1+\alpha z)\,\frac{\partial^2 v_3}{\partial x^2} = d_1(1+\alpha z)\,\frac{\partial^2 v_3}{\partial t^2}, \tag{1.12}$$

where $d_1 = \rho_{11} - \rho_{12}^2/\rho_{22}$.

For SH-waves, the solution of Eq. (1.12) can be written as

$$v_3(x,z,t) = f_3(z)\,\mathrm{e}^{i(kx-wt)}\,. \tag{1.13}$$

Equations (1.12) and (1.13) yield the following equation for the function $f_3(z)$:

$$\frac{d^3 f_3(z)}{dz^2} + \frac{\alpha}{1+\alpha z}\,\frac{df_3(z)}{dz} - k^2\Big(1 - \frac{P_2}{2N_3} - \frac{c^2}{\beta_3^2}\Big)f_3(z) = 0 \tag{1.14}$$

($\beta_3 = \sqrt{N_3/d_1}$ is the shear wave velocity in the lower poroelastic half-space). Using the dimensionless parameters $\gamma_{11} = \rho_{11}/\rho$, $\gamma_{12} = \rho_{12}/\rho$, and $\gamma_{22} = \rho_{22}/\rho$ [21], one can write the expression for β_3

$$\beta_3 = \sqrt{N_3/d_1} = \beta_0\sqrt{1/d},$$

where $\beta_0 = \sqrt{N_3/\rho}$, $\rho = \rho_{11} + 2\rho_{12} + \rho_{22}$, and $d = \gamma_{11} - \gamma_{12}^2/\gamma_{22}$ is the porosity parameter.

Thus, the following variations of d are possible:
(1) if the lower half-space is a non-porous solid, then $d \to 1$;
(2) if the lower half-space is a fluid, then $d \to 0$;.
(3) is the lower half-space is a poroelastic medium, then $0 < d < 1$.
After the replacement $f_3(z) = \varphi(z)/\sqrt{1+\alpha z}$, Eq. (1.14) takes the form

$$\varphi''(z) + \Big[\frac{\alpha^2}{4(1+\alpha z)^2} - k^2\Big(1 - \frac{P_2}{2N_3} - \frac{c^2 d}{\beta_0^2}\Big)\Big]\varphi(z) = 0. \tag{1.15}$$

Introducing a new independent variable $\eta = 2Fk(1+\alpha z)/\alpha$ in Eq. (1.15), we obtain

$$\frac{d^2\varphi(\eta)}{d\eta^2} + \Big(\frac{1}{4\eta^2} + \frac{R}{\eta} - \frac{1}{4}\Big)\varphi(\eta) = 0, \tag{1.16}$$

where

$$R = \frac{k}{2F\alpha}\Big(\frac{c^2}{\beta_3^2} - 1 + \frac{P_2}{2N_3} + F^2\Big), \qquad F = \sqrt{1 - \frac{P_2}{2N_3} - \frac{c^2 d}{\beta_0^2}}\,.$$

Equation (1.16) is the well-known Whittaker differential equation [27] whose solution is

$$\varphi(\eta) = A_5 W_{(R,0)}(\eta) + A_6 W_{(-R,0)}(\eta)$$

(A_5 and A_6 are arbitrary constants; $W_{(R,0)}(\eta)$ and $W_{(-R,0)}(\eta)$ are the Whittaker functions).

As the displacement vanishes as $z \to \infty$, then

$$\varphi(\eta) = A_5 W_{(R,0)}(\eta).$$

Therefore, we have

$$f_3(z) = \frac{\varphi(\eta)}{\sqrt{1+\alpha z}} = \frac{A_5 W_{(R,0)}(\eta)}{\sqrt{1+\alpha z}}\,;$$

$$v_3(x,z,t) = \frac{A_5 W_{(R,0)}(\eta)}{\sqrt{1+\alpha z}}\ \mathrm{e}^{i(kx-wt)}\,. \tag{1.17}$$

2. BOUNDARY CONDITIONS AND FREQUENCY EQUATION

The interface boundary conditions are formulated as follows.

1) At $z = -H$, the displacements are continuous, i.e., $v_1 = v_2$, and the stresses are also continuous, i.e., $(\sigma_{yz})_1 = (\sigma_{yz})_2$; therefore,

$$L_1\Big(\frac{\partial v_1}{\partial z}\Big)\Big|_{z=-H} = C_{44}\Big(\frac{\partial v_2}{\partial z}\Big)\Big|_{z=-H}; \tag{2.1}$$

2) At $z = 0$, the displacements are continuous, i.e., $v_2 = v_3$, and the stresses are also continuous, i.e., $(\sigma_{yz})_2 = (\sigma_{yz})_3$; therefore,

$$C_{44}\Big(\frac{\partial v_2}{\partial z}\Big)\Big|_{z=0} = N_3(1+\alpha z)\Big(\frac{\partial v_3}{\partial z}\Big)\Big|_{z=0}. \tag{2.2}$$

Substituting Eqs. (1.3), (1.9), and (1.17) into Eqs. (2.1)–(2.2) and expanding the Whittaker function and its derivative up to the linear term, we obtain four equations for four unknown constants A_i $(i = 1,2,3,4)$. The condition of existence of the nontrivial solution of this system yields the relation

$$\tan\,(qH) = \frac{qC_{44}(L_1 S_1 X_1 - N_3 X_2)}{X_1 C_{44}^2 b^2 + C_{46} X_1 k^2 + N_3 X_2 S_1 L_1}, \tag{2.3}$$

where

$$X_1 = \mathrm{e}^{-Fk/\alpha}\ \Big(\frac{\alpha}{8FK} - 1\Big), \qquad X_2 = \frac{-X_1(1+\alpha)}{2} - \mathrm{e}^{-Fk/\alpha}\ \frac{\alpha^2}{16F^2k^2}.$$

Relation (2.3) is the frequency equation for the SH-wave in the triclinic layer.

Consider particular cases of the frequency equation.

Case 1: If $N_1 = L_1$ (i.e., the upper half-space is isotropic and elastic), $P_2 = 0$, $d \to 1$, and $\alpha = 0$, (i.e., the lower half-space is homogeneous, stress-free, isotropic, and elastic), Eq. (2.3) takes the form

$$\tan\,(qkH) = \frac{C_{44}q(N_1S_1 + N_3Fk)}{C_{44}^2b^2 + C_{46}k^2 - N_1N_3S_1Fk}\,. \tag{2.4}$$

Case 2. If, in addition to the conditions of Case 1, $S_1 = 0$ (i.e., there is no upper half space), then the frequency equation (2.4) reduces to

$$\tan\left(\sqrt{\frac{P_1}{2C_{44}} + \frac{c^2}{\beta_2^2} + \frac{C_{46}^2}{C_{44}^2} - \frac{C_{66}}{C_{44}}}\,kH\right) = \frac{N_3C_{44}\sqrt{P_1/(2C_{44}) + c^2/\beta_2^2 + C_{46}^2/C_{44}^2 - C_{66}/C_{44}}\,\sqrt{1 - c^2/\beta_3^2}}{C_{44}^2(P_1/(2C_{44}) + c^2/\beta_2^2 - C_{66}/C_{44}) + C_{46}}\,. \tag{2.5}$$

Equation (2.5) represents the dispersion equation for the SH-wave in the initially stressed triclinic layer lying over a homogeneous elastic half-space.

Case 3: If, in addition to the conditions of Case 2, $P_1 = 0$, $C_{46} = 0$, and $C_{44} = C_{66} = N_2$, then Eq. (2.5) reduces to

$$\tan\left[\left(\frac{c^2}{\beta_2^2} - 1\right)^{1/2} kH\right] = \frac{N_3}{N_2}\,\frac{(1 - c^2/\beta_3^2)^{1/2}}{(c^2/\beta_2^2 - 1)^{1/2}}\,. \tag{2.6}$$

Here $\beta_2 < c < \beta_3$, $\beta_2 = \sqrt{N_2/\rho_2}$, and $\beta_3 = \sqrt{N_3/\rho_{11}}$.

Equation (2.6) represents the frequency equation of the classical Love wave in a homogeneous layer lying over a homogeneous half-space [1].

3. NUMERICAL RESULTS

The phase velocity against the wave number is computed numerically by using the bisection method implemented in the MATLAB software, and the results are depicted in Figs. 2–5. The numerical calculations are performed with the following data: $N_1 = 6.7 \cdot 10^{10}$ N/m^2, $L_1 = 3.96 \cdot 10^{10}$ N/m^2, and $\rho_1 = 7140$ kg/m^3 [4] for the upper transversely isotropic half-space (zinc); $C_{44} = 5.64 \cdot 10^9$ N/m^2, $C_{46} = 0$, $C_{66} = 6.912 \cdot 10^9$ N/m^2, and $\rho_2 = 2400$ kg/m^3 [8] for the triclinic layer (Vosges Sandstone); $N_3 = 0.922 \cdot 10^{10}$ N/m^2, $\rho_{11} = 1.9032 \cdot 10^3$ kg/m^3, $\rho_{12} = 0$, and $\rho_{22} = 0.268 \cdot 10^3$ kg/m^3 [25] for the lower anisotropic poroelastic half-space (sandstone saturated with water); the remaining parameter values are $P_1/(2C_{44}) = 0.75$, $P_2/(2N_3) = 0.2$, $d = 0.2$, and $\alpha H = 15$.

Figure 2 illustrates the effect of the initial stress $P_1/(2C_{44})$ of the triclinic layer on the phase velocity c_2/β_2 as a function of the wave number kH. It is noticed that the phase velocity decreases with an increase in the initial stress and wave number.

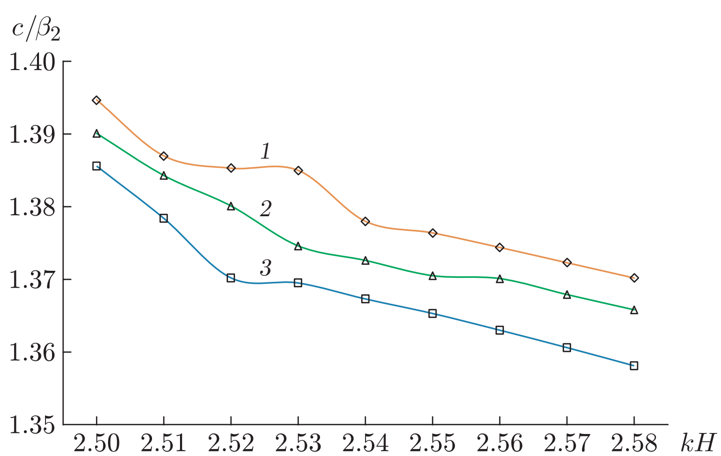

Fig. 2. Phase velocity versus the wave number for different values of the initial stress $P_1/(2C_{44})$ in the triclinic layer: $P_1/(2C_{44}) = 0.71$ (*1*), 0.73 (*2*), and 0.75 (*3*).

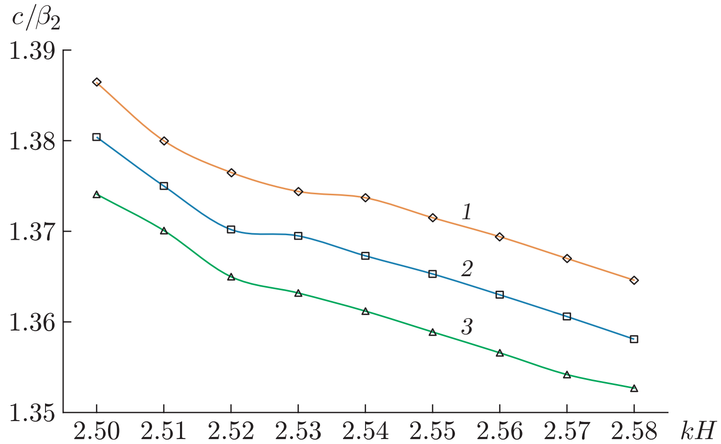

Fig. 3. Phase velocity versus the wave number for different values of the stress $P_2/(2N_3)$ in the lower half-space: $P_2/(2N_3) = 0.16$ (*1*), 0.20 (*2*), and 0.24 (*3*).

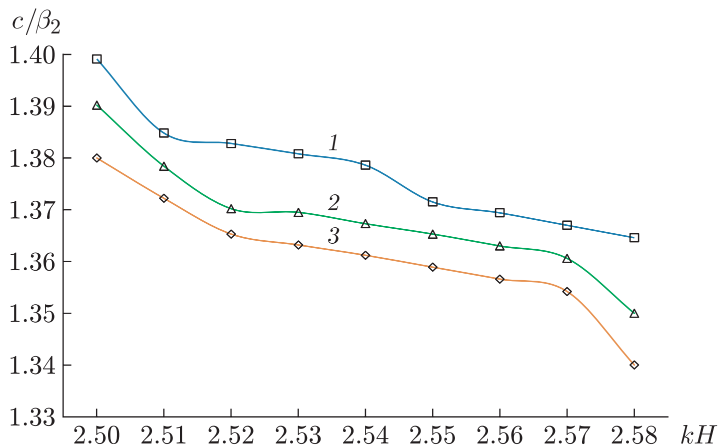

Fig. 4. Phase velocity versus the wave number for the SH-waves for different values of porosity d: $d = 0.15$ (*1*), 0.20 (*2*), and 0.25 (*3*).

Figure 3 shows the effect of the initial stress $P_2/(2N_3)$ of the lower half-space on the phase velocity as a function of the wave number. It can be seen from Fig. 3 that the phase velocity decreases as the initial stress increases.

The effect of the porosity parameter d on dispersion of the SH-wave is shown in Fig. 4. It is observed that the phase velocity diminishes with an increment in the porosity parameter.

Figure 5 displays the effect of the heterogeneity parameter αH on the phase velocity as a function of the wave number. It is seen from Fig. 5 that the phase velocity decreases with an increase in the heterogeneity parameter αH.

CONCLUSIONS

SH-wave propagation in an initially stressed triclinic layer sandwiched between a transversely isotropic elastic half space and a heterogeneous poroelastic half-space is studied. The solutions for the displacements in the layer and half-spaces are obtained separately in closed form. For the heterogeneous poroelastic half-space, the solution is expressed as the Whittaker function. Significant effects of the initial stress, porosity, and inhomogeneity of individual media are detected. The following results are the outcomes of this study. The phase velocity in general decreases with an increase in the dimensionless wave number in all cases. In the case with initial stresses, the phase velocity

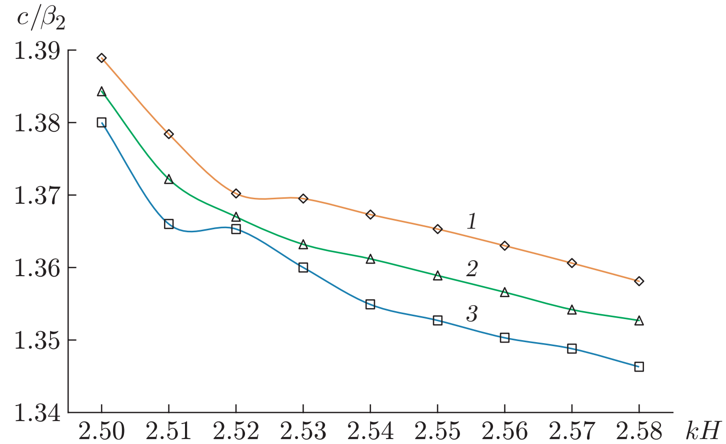

Fig. 5. Phase velocity versus the wave number for different values of the heterogeneity parameter αH: $\alpha H = 15.0$ (*1*), 15.5 (*2*), and 16.0 (*3*).

decreases as the heterogeneity parameter increases and increases as the porosity increases. If the upper half-space, initial stress, and heterogeneity are removed, the dispersion relation coincides with that of the classical Love waves.

The results obtained in the study can help in modeling the exploration of natural resources, mining crystals, and minerals inside the Earth.

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STUDY OF DISPERSIVE BEHAVIOR OF WAVES IN POROELASTIC LAYERS AND HALF SPACES

Thesis

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for the Award of the Degree of

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in
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Certificate

This is to certify that the thesis entitled "**Study of Dispersive Behavior of Waves in Poroelastic Layers and Half Spaces**" being submitted to Kakatiya University, Warangal by **M.Venu Gopal** in partial fulfilment of the requirements for the award of the degree of **Doctor of Philosophy** in **Mathematics**, is a bonafide record of the work carried out by him under my guidance and supervision.

The results presented in this thesis have been verified, and are found to be satisfactory. The results embodied in this thesis have not been submitted to any other University for the award of any other degree or diploma.

Date: 30/12/21

P. M. Reddy

(Prof. P. Malla Reddy)
Advisor

Declaration

I hereby declare that the work which is being presented in this thesis entitled **"Study of Dispersive Behavior of Waves in Poroelastic Layers and Half Spaces"** submitted towards partial fulfilment of the requirements for the award of the degree of **Doctor of Philosophy** in **Mathematics** is an authentic record of my own work carried out under the supervision of **Prof. P. Malla Reddy**, Department of Mathematics, Kakatiya University.

To the best of my knowledge and belief, this work is no resemblance with any other material previously published except where due reference has been cited in text.

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